STABILITY ESTIMATE IN SCATTERING THEORY AND ITS APPLICATION TO MESOSCOPIC SYSTEMS AND QUANTUM CHAOS

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Abstract

We consider scattering of a free quantum particle on a singular potential with rather arbitrary shape of the support of the potential. In the classical limit $\hbar = 0$ this problem reduces to the well known problem of chaotic scattering. The universal estimates for the stability of the scattering amplitudes are derived. The application of the obtained results to the mesoscopic systems and quantum chaos are discussed. We also discuss a possibility of experimental verification of the obtained results.

I. Introduction

Recently much attention has been paid to the theoretical and experimental investigations of the scattering of a free quantum particle on the obstacles with rather complicated form of boundaries. Of special interest are the studies of the scattering processes in mesoscopic systems at the ballistic regime when quantum effects and the geometry of the scattering potential are important [1-13]. Usually, these quantum systems are nonintegrable, and if they are treated classically they exhibit dynamical chaos, that is, strong (exponential) instability of motion under small variation of parameters (such as energy of an incident wave, form of the potential, etc). That is why one of the main problems in studying such systems is to determine the role and contribution of fluctuations and correlations in the scattering amplitudes and cross sections [14-22].

In this paper we consider a scattering problem for a free quantum particle scattered by a bounded obstacle with rather arbitrary shapes of the boundary. The boundary may consist of several connected components. Similar situation occurs in the processes of ballistic scattering in the mesoscopic systems widely considered nowadays [1-13]. The results obtained can be formulated in the following way. It is shown that there exists the region of parameters where small variation of rather arbitrary singular potential (note, that in this case the variation of the whole energy is infinite) leads only to small variations of the scattering amplitudes. This region of parameters can be defined as a region of strong correlations. These correlations are universal, and do not depend on the concrete structure of the resonances. We discuss the obtained results in connection with the general problem of quantum chaos and experimental observations of fluctuation and correlation effects in quantum

chaotic scattering. The paper is organized as follows. In section 2 we present a stability estimate for the scattering amplitudes for rather wide classes of potentials. In section 3 a proof of the stability of the scattering amplitudes is given for a singular potential. Applications to the quantum chaotic scattering are discussed in section 4.

2. Stability Estimate for the Scattering Amplitude

In this section we prove that small variations of the potential lead to small perturbations of the scattering amplitude for a class of strongly singular potentials which can take infinite values on sets of positive measure. The notion of small variations will be specified.

1. Let $D = \bigcup_{j=1}^{J} D_j$, $\Gamma := \partial D = \bigcup_{j=1}^{J} \Gamma_j$, $D_j \subset R^n$ is a bounded domain with a $C^{2,\nu}$, $0 < \nu \leq 1$, boundary Γ_j. This means that in the local coordinates the equation of $\Gamma_j := \partial D_j$ is $x_n = \phi(x')$, $x' := (x_1, x_2, ..., x_{n-1})$, $\phi \in C^{2,\nu}$, $||\phi||_{C^{2,\nu}} \leq \Phi_\nu$.

Assume $D \subset B_a := \{x : |x| \leq a\}$, and $D_j \bigcap D_i = \emptyset$ if $i \neq j$, $J < \infty$. Define $u_0 := \exp(ik\alpha \cdot x)$. Define

$$q(x;t) := t\chi_D(x), \quad \chi_D(x) := \begin{cases} 1, & \text{in } D, \\ 0, & \text{in } D' := R^n \setminus D, \end{cases}$$

where parameter $t \in [1, \infty]$. For definiteness take only $n = 3$ in what follows. Consider the scattering problem

$$[\nabla^2 + k^2 - q(x;t)]u = 0 \quad in \quad R^3. \tag{1}$$

$$u = \exp(ik\alpha \cdot x) + A^{(t)}(\alpha', \alpha, k)\frac{\exp(ikr)}{r} + o\left(\frac{1}{r}\right), \tag{2}$$
$$r := |x| \to \infty, \quad \frac{x}{|x|} := \alpha'.$$

The scattering solution $u(x,\alpha,k;t):=u(t)$ is uniquely defined as the solution of (1), (2). It was proved in [26-28], that

$$|u(t)-u_\Gamma|\to 0, \quad as \quad t\to+\infty, \tag{3}$$

where u_Γ is the scattering solution to the obstacle scattering problem

$$(\nabla^2+k^2)u_\Gamma=0 \quad in \quad D', \quad u_\Gamma=0 \quad on \quad \Gamma, \tag{4}$$

$$u_\Gamma=u_0+A_\Gamma(\alpha',\alpha,k)\frac{\exp(ikr)}{r}+o\left(\frac{1}{r}\right), \quad r=|x|\to\infty, \quad \alpha':=\frac{x}{r}. \tag{5}$$

The relation (3) has the following meaning

$$||u(t)||_{L^2(D)}\le\frac{c}{\sqrt{t}}, \quad ||u(t)-u_\Gamma||_{H^2(\tilde{D}')}\le\frac{c}{t^{1/4}}, \tag{6}$$

$$||u(t)||_{L^2(\Gamma)}\le\frac{c}{t^{1/4}}, \tag{7}$$

where $\tilde{D}'$ is any compact strictly inner subdomain of D'. Here and below $c>0$ denote various positive constants independent of t or other parameters which vary.

Estimates (6),(7) are proved in [26-28]. It is proved in [24] that if $q_j(x)$, $j=1,2$, generate the scattering amplitudes $A_j(\alpha',\alpha,k)$, then, the following relation holds

$$-4\pi A(\alpha',\alpha,k)=\int p(x)u_1(x,\alpha,k)u_2(x,-\alpha',k)dx, \tag{8}$$

where

$$A:=A_1-A_2, \quad p:=q_1-q_2, \tag{9}$$

and u_j is the scattering solution corresponding to q_j. Formula (8) is derived in [24] under the assumption that $q_j(x)\in L^p_{loc}$, $p>n/2$, and $q(x)$ is in $L^1(B'_R)$, where $B'_R:=R^3\setminus B_R$, $B_R:=\{x:|x|\le R\}$, $R>0$ is an arbitrary large fixed number.

In [29] an analog of (8) is derived for obstacle scattering. Namely, it is proved in [29] that if Γ_j, $j = 1, 2$, are bounded sufficiently smooth (say, Lipschitz) surfaces, and A_j are the corresponding scattering amplitudes, $A_j := A_{\Gamma_j}$, $A := A_1 - A_2$, then [29, formula (4)]

$$-4\pi A(\alpha', \alpha, k) \tag{10}$$

$$= \int_{\Gamma_{12}} [\bar{u}_{1N}(s, \alpha, k) u_2(s, -\alpha', k) - u_1(s, \alpha, k) u_{2N}(s, -\alpha', k)] ds,$$

where N is the exterior unit normal to $\Gamma_{12} = \partial D_{12}$, where $D_{12} := D_1 \bigcup D_2$.

2. We claim that, uniformly in $t_j \in [1, \infty]$, the following stability estimate holds

$$\sup_{\alpha', \alpha \in S^2; 0 < k_1 \leq k \leq k_2 < \infty} |A_{D_1}^{(t_1)}(\alpha', \alpha, k) - A_{D_2}^{(t_2)}(\alpha', \alpha, k)| \tag{11}$$

$$\leq c\{[min(t_1, t_2)]^{-1/4} + \rho(D_1, D_2)\},$$

where $c = const. > 0$, c is independent on $t_j \in [1, \infty]$, and on $D_j \subset B_a$, $j = 1, 2$, such that $||\phi_j||_{C^{2,\nu}} \leq \Phi_\nu$.

The distance $\rho(D_1, D_2)$ in (11) is defined by the formula

$$\rho(D_1, D_2) := \sup_{x \in \partial D_1} \inf_{y \in \partial D_2} |x - y|$$

3. Note, that if $t \in [1, t_0]$, where $1 < t_0 < \infty$ is any fixed number, then the following estimate can be derived from (8)

$$\sup_{\alpha', \alpha \in S^2; 0 < k_1 \leq k \leq k_2 < \infty} |A_{D_1}^{(t_1)}(\alpha', \alpha, k) - A_{D_2}^{(t_2)}(\alpha', \alpha, k)|$$

$$\leq \frac{c}{4\pi} |t_1 - t_2| \int_{D_1 \bigcap D_2} dx + \frac{ct_0}{4\pi} \int_{D_{12} \setminus (D_1 \bigcap D_2)} dx$$

$$\leq \frac{c}{4\pi} |t_1 - t_2| |D_1 \bigcap D_2| + \frac{ct_0}{4\pi} \{|\partial D_1| + |\partial D_2|\} \rho(D_1, D_2)$$

$$\leq c\{|t_1 - t_2| + \rho(D_1, D_2)\}. \tag{12}$$

Here we have used the known estimate [24], [25]

$$\max_{x\in R^3;\alpha\in S^2;0<k_1\le k\le k_2<\infty} |u_j| \le c. \tag{13}$$

In (12) $|\partial D_j|$ denotes the area of the surface ∂D_j, and $|D_1 \bigcap D_2|$ denotes the volume of $D_1 \bigcap D_2$.

4. If $t_1 = t_2 = +\infty$, then, the stability estimate

$$\sup_{\alpha',\alpha\in S^2;0<k_1\le k\le k_2<\infty} |A_1(\alpha',\alpha,k) - A_2(\alpha',\alpha,k)| \le c\rho(D_1,D_2) \tag{14}$$

follows from formula (10), since

$$\sup_{s\in\Gamma_j;\alpha\in S^2;0<k_1\le k\le k_2<\infty} |u_{jN}(s,\alpha,k)| \le c,$$

$$\sup_{\alpha'\in S^2;s\in\Gamma_{j+1};0<k_1\le k\le k_2<\infty} |u_j(s,-\alpha',k)| \le c\rho(D_1,D_2).$$

Here $\Gamma_3 := \Gamma_1$, $j = 1, 2$.

The basic result (11), which contains both stability estimates (12) and (14), is of interest because the inequality (11) holds *uniformly in* t, $t \in [1,\infty]$.

5. As an example, we present here the results on the dependence $c(k)$ in (14) for the special case of the scattering potential. We claim that the constant c in (14) is of the order $O(k^2)$ as k goes to infinity, under the following assumptions: i) $J = 1$, ii) $s \cdot N > b > 0$ for s in S_1 ($S_1 := \Gamma$) and for s in the perturbed surface, say S_2; here N is the outer normal to S_1 (or S_2) at the point $s, b > 0$ is a constant independent of s, k and other parameters.

Proof of the claim: If ii) holds, then from the estimate (6) in [23,p.66] it follows that $||v||_{B_R} < c$, c is always assumed to be independent of k, $v := u - u_0$, where u is the scattering solution corresponding to S_1, and u_0 is the plane wave. From this and the Helmholtz equation one gets $||v||_2 < ck^2$, where $||v||_2$ is the Sobolev space H^2 norm. Let $|v_N|$ stay for the $L^2(S_1)$ norm

of v_N on S_1. Then, an interpolation inequality yields the desired estimate: $|v_N| < ck^{3/2}$. This estimate implies the claim that the constant c in (14) is of the order $O(k^2)$ as k grows to infinity. Indeed, estimating integrals in (10) by Cauchy's inequality one gets the sum of the products of the terms of the type $|v_N|\ |v|$ and terms of lower order in k which are easy to estimate by $O(k^{3/2})$. By an interpolation inequality, the norm $|v|$ is $O(k^{1/2})$, so the result follows. Let us formulate the known interpolation inequalities used above (see [28])

$$||D^r v||_{L^2(S_1)} < ct^{3/2-r}||v||_2 + t^{-1/2-r}||v||, \tag{15}$$

where $||v||$ is the L^2 norm in $B_a \setminus D, \partial D = S_1$, $t > 0$ in (15) is an arbitrary parameter, and $r = 0$ or 1. Take $r = 0$ in (15) and minimize in $t > 0$ the right-hand side of (15), using the formulas $||v||_2 < ck^2$, $||v|| < c$, to get for the right-hand side the estimate $O(k^{1/2})$. Similar argument for $r = 1$ yields the estimate $O(k^{3/2})$ as claimed.

Remark. The order in k as $k \to \infty$ in the estimate for the constant c in (14), is not optimal. The optimal order is probably $O(1)$. For a ball, for instance, we can prove that $|v_N| = O(k)$, rather than $O(k^{3/2})$ and $|v| = O(1)$, rather than $O(k^{1/2})$. This yields $c = O(k)$ as $k \to \infty$. The estimate based on the Cauchy inequality, used in our derivation, does not take into account possible cancellations during integration in (10) due to oscillations of the integrand for large k. The optimal orders are: 1) $O(1)$ for $|v|$, 2) $O(k)$ for $|v_N|$, and 3) $O(1)$ for the cross section as $k \to \infty$. These conclusions can be also obtained from the geometrical optics approximation (see formula (150.16) in H.Honl,A.Maue, K.Westpfahl, Theorie der Beugung, Springer Verlag, Berlin, 1961).

6. Let us formulate the result proved in [28].

Theorem 1. Under the assumption made in section 2.1, estimate (11) holds

with the constant $c > 0$ independent of t, where $t \in [1, \infty]$, $D_j \subset B_a$, $\partial D_j \subset C^{2,\nu}$, $||\phi_j||_{C^{2,\nu}} \leq \Phi_\nu$.

In section 3 the proof of estimate (14) is given for the case $t_1 = t_2 = \infty$ which is of interest in applications. In section 4 applications are discussed.

3. Proof of the Stability Estimate (14)

Let us assume that

$$q_j(x) = \begin{cases} +\infty, & \text{in } D_j, \\ 0, & \text{in } D_j' := R^n \setminus D_j, n \leq 2. \end{cases} \tag{16}$$

This is the case discussed in section 2.4 (see formula (14)). We assume $n = 3$ for definiteness. The argument is the same for $n \geq 1$.

There are three ways to prove estimate (14) under the assumption (16). One way is to take $t_1 = t_2 = +\infty$ in (11), and note that the right-hand side equals $c\rho(D_1, D_2)$ if $t_1 = t_2 = +\infty$. The second way, is to take $t_1 = t_2 = t < \infty$, and then let $t \to +\infty$, and use formula (8) and estimates (6), (7). These estimates allow one to derive formula (10) from which estimate (14) follows. Estimate (14) is a particular form of estimate (11) for the case when $\min(t_1, t_2) = +\infty$. The third way is based on estimate (10). Let us use this way. We assume that the distance $\rho(D_j, D_m)$, $j \neq m$, is much greater than the distance $\rho(D_j, \tilde{D}_j)$, where $\tilde{D}_j$ is the perturbed domain D_j. The number J of the connected components of the domain D is fixed and finite. Therefore, the input of the variation of ∂D in the scattering amplitude is of the order of magnitude of the input of the variation of ∂D_j , $1 \leq j \leq J$. Therefore, one may use formula (10) assuming that ∂D has one connected component ∂D_1, and $\partial D_2 := \partial \tilde{D}_1$ is a small variation of ∂D_1 in the sense that $\rho(D_1, D_2)$

is small. It follows from (10) that

$$|A(\alpha',\alpha,k)| \leq \frac{1}{4\pi}\int_{\Gamma_1'} |u_{1N}(s,\alpha,k)u_2(s,-\alpha',k)|ds \tag{17}$$

$$+\int_{\Gamma_2'} |u_1(s,\alpha,k)u_{2N}(s,-\alpha',k)|ds := I_1 + I_2,$$

where Γ_1' is the part of Γ_1 which lies outside D_2, and Γ_2' is the part of Γ_2 which lies outside D_1.

One can use the following estimates

$$\gamma := \max_{j=1,2} \sup_{s\in\Gamma_j;\beta\in S^2;0<k_1\leq k\leq k_2<\infty} |u_{jN}(s,\beta,k)| \leq c, \tag{18}$$

$$\max_{j=1,2} \sup_{s\in\Gamma_j';\beta\in S^2;0<k_1\leq k\leq k_2<\infty} |u_{j+1}(s,\beta,k)| \leq c\rho(D_1,D_2), \quad u_3 := u_1, \tag{19}$$

and formula (17), to get the desired estimate (14). Let us discuss estimates (18) and (19). The constant c in (18) and (19) depends on the parameters k_1, k_2, a, and on the parameter Φ_ν, which is introduced in section 2.1, and which describes the smoothness of the boundary: $||\phi_j||_{C^{2,\nu}} \leq \Phi_\nu$. This constant does not depend on the particular choice of D_j. Let us prove the last claim. Suppose on the contrary, that there exists a sequence D_{jn} of the obstacles $D_{jn} \subset B_a$, $||\phi_{jn}||_{C^{2,\nu}} \leq \Phi_\nu$, such that $\gamma_n \geq c_n$, $c_n \to \infty$, where c_n are the constants in (18), (19), and γ_n is γ for the obstacle D_{jn}, $n = 1, 2, ...$. By the Arzela-Ascoli compactness theorem one can assume that

$$\phi_{jn} \stackrel{C^{2,\nu'}}{\to} \psi_j, \quad 0 < \nu' < \nu, \quad u_{jn} \stackrel{H^2_{loc}}{\to} u_j, \quad n \to \infty,$$

where u_j is the scattering solution corresponding to the limiting configuration of the surfaces Γ_1, Γ_2. For fixed surfaces Γ_1 and Γ_2, estimates (18) and (19) hold [23].

Note that it is sufficient to prove estimate (18). Indeed,

$$|u_1(s,\beta,k)| = |u_1(s,\beta,k) - u_1(\tilde{s},\beta,k)| \leq \sup |u_{1N}(s,\beta,k)||s-\tilde{s}|$$

$$\leq c\rho(D_1, D_2),$$

where $s \in \Gamma_2'$, $\tilde{s} \in \Gamma_1$, $u_1(\tilde{s}, \beta, k) = 0$, and the segment $\tilde{s}s$ is directed along the normal to Γ_2'. A similar argument is valid for $u_2(s, \beta, k) = 0$, $s \in \Gamma_1'$.

If $\Gamma_{jn} \to \Gamma_j$ in the sense $\phi_{jn} \overset{C^{2,\nu'}}{\to} \psi_j$ as $n \to \infty$), then $u_{jNn} \to u_{jN}$ as $n \to \infty$ (uniformly in $s \in \Gamma_j$ and in the parameters $\beta \in S^2$, $k \in [k_1, k_2]$, $0 < k_1 < k_2 < \infty$), so that $\gamma_n \to \gamma$ as $n \to \infty$. Here γ is the number defined by the left-hand side of (18) with u_j corresponding to the limiting surfaces Γ_j. Since this $\gamma < \infty$, one obtains a contradiction: the inequality $\gamma_n \geq c_n \to +\infty$ contradicts to the equation $\gamma_n \to \gamma < \infty$. This contradiction proves that the constant c in (18) and (19) does not depend on the particular choice of the obstacles D_j as long as the two conditions are satisfied: $D_j \subset B_a$, $||\phi_j||_{C^{2,\nu}} \leq \Phi_\nu$, and the parameters a, Φ_ν, k_1 and k_2 define the value of c in (18), (19) and (14).

4. Appications to the Chaotic Scattering

The results derived above have direct application to the so-called chaotic scattering [14-22,30-32]. The problem of fluctuations of the scattering amplitudes and cross sections in the processes of elastic (and inelastic) collisions is well known, and has a long history (see [33-38] and references therein). In the elastic scattering which was considered in sections 1-3, these fluctuations of the scattering amplitudes can appear because of a high sensitivity to the details of the scattering: the parameters of the incident wave and the geometry of the scatter potential. At the same time, the coherent effects (correlations) are also present in the scattering processes in some region of parameters [21,22,33,34,39]. Thus, the problem arises: how does one sep-

arate and describe the random and the coherent effects in the scatttering processes, and how does one measure their contribution in experiments?

The first theoretical investigations of the statistical properties (fluctuations) of the scattering amplitudes and cross sections were presented in [33-38] (Ericson fluctuations). According to [33,34], the main reasons why the scattering amplitudes become random are the following. Let an incident wave (the first term in (2)) have a wave-length $\lambda = 2\pi/k$ much smaller than the characteristic dimension L of the region D where the scattering potential $q(x)$ in (1) is located: $kL \gg 1$. Before escaping from the region D, the incident wave can be reflected a large number of times from the boundaries Γ_j of the support of the potential $q(x)$. In this case, a wave close to a standing wave appears in the system. These "quasi-standing" (or quasy-stationary) waves can be associated with the resonances in the scattering amplitude. Each n-th resonance is characterized by two main parameters: the energy E_n, and the width Γ_n [40]. There is usually one more important parameter which characterizes the spacing between the neighboring resonances: ΔE_n. Because the process of scattering is completely defined, the scattering amplitudes should be reproducible in different experiments, provided that all conditions remain identical. However, as was mentioned above, under the condition $kL \gg 1$ the number of reflections of the incident wave in the region D can be very large (in [33,34] also the following inequality is assumed to be satisfied: $\Gamma_n/\Delta E_n \gg 1$, which is called the regime of overlapping levels). Then, a small variation of parameters can completely change the "trajectory" of the wave, and consequently, the phase of the scattering amplitude. These ideas were developed in [33,34,36] on the basis of the statistical approach [41].

Recently, the problem of fluctuations of the scattering amplitudes has

attracted additional interest in connection with the so-called "chaotic (irregular) scattering" (CS) in chemical reactions, particle physics, mesoscopic systems and other areas of physics [14-22,30-32]. The investigations of the CS can be conventionally divided into three groups: (1) classical CS (CCS), (2) semiclassical CS (SCS), (3) quantum CS (QCS). The basic ideas are associated with the CCS, since only in this case the dynamical chaos occurs. The investigations of the CCS were stimulated by the significant progress achieved recently in studying of the dynamical chaos in the classical bounded Hamiltonian systems [42-45]. The classical phase space in this case can be very complicated, and each of the trajectories belongs to one of the following three classes: (a) stable periodical trajectories, (b) unstable periodical trajectories, (c) chaotic (unperiodical) trajectories. Dynamical chaos in bounded systems is stationary in the sense that it does not disappear at large times ($T \to \infty$). The systems where the CCS takes place are unbounded, and the additional trajectories appear: (d) unbounded trajectories. In the case of a singular potential $q(x)$ considered above the trajectories (a) can be absent (see, for example, [15]), and the trajectories (b) and (c) represent a "repeller" Ω_R [15]. For the trajectories (d) this repeller leads to the "transient chaos" which was previously investigated in various bounded conservative and dissipative systems (see, for example, [46-48]).

The main achievements in the CCS are associated with the understanding of the following facts: (1) although, in the CCS a direct contribution in the cross section is connected with the trajectories (d), the influence of the repeller Ω_R - bounded (trapped) trajectories on the process of scattering and fluctuations plays a very important role; (2) the CCS is a general phenomenon rather then an exception. (In some special cases of a singular potential [49], the set Ω_R can consist of only one unstable periodic trajectory). Usually,

for singular potentials considered above, a repeller Ω_R is a Cantor set with a fractal structure (see, for example, paper [15] where an elastic scattering on three hard discs (3HD) was considered), and is characterized by several quantities, such as the Hausdorff dimension D_H, Lyapunov exponents λ_i, the Kolmogorov-Sinai entropy per unit time h_{KS}, the escape rate γ, and other quantities (see [15] and references therein). There are some relations between these parameters, for example, (see [15]):

$$\gamma = \sum_{\lambda_i > 0} \lambda_i - h_{KS}. \tag{23}$$

The escape rate γ is a classical equivalent of the resonance width Γ: $\gamma \sim \Gamma/\hbar$ [15]. So, the relation (23) shows a fundamental property of the CCS: when a repeller Ω_R is chaotic ($h_{KS} > 0$), the escape rate (and the resonance width Γ) is decreasing. Also, in this case large fluctuations appear in the quantities which characterize the process of CCS, for example, in the time delay function [15,20,22].

When one investigates the SCS and the QCS, the main problem is: what are the "fingerprints" of the classical chaos on the quantum scattering ? For the first time, the problem of QCS was considered in [14], where the elastic scattering was studied on a two-dimensional surface of a constant negative curvature. According to [14], the scattering phase shift as a function of the momentum is given by the phase angle of the Riemann's zeta function, and displays a very complicated (chaotic) behavior (see for details [14,21,22]). In [16] the SCS was studied in the system of 3HD using the analysis based on the Gutzwiller trace formula [50]. This trace formula is valid when all periodic orbits of the repeller Ω_R are unstable and isolated. Both these conditions can be satisfied for the singular potential $q(x, t \to \infty)$ considered in sections 1-3, including a particular case of a singular potential of the 3HD system

considered in [15-18].

The quantum analysis presented in [21,22] shows that in the QCS the statistical properties of the fluctuations in the cross section can be described by the theory of random matrix ensembles [41]. Different aspects on the problem of fluctuations in the SCS and QCS are discussed in [14,16-19,21-39].

At the same time, much less is known about the contribution and characterisic properties of the correlations (coherent component) in the chaotic scattering. As was pointed out in [33,34], a significant level of correlations in the cross section should be expected when, for example, the energy change δE of the incident wave in (2) is small compared with the resonance width Γ ($\Gamma/\delta E > 1$). According to [33,34], in this case essentially the same states are exited, and the scattering amplitudes are changed insignificantly. The existence of correlations in the QCS was discussed also in [21,22] for some quasi-1D periodical potential (in [22] also an experiment is discussed in connection with the correlations in the chaotic scattering). It was shown in [21,22] that the correlations in energy for the matrix elements of the S-matrix exist, and exhibit themselves when $\Gamma/\delta E > 1$, in agreement with the Ericson hypothesis [33,34].

In connection with the problem of correlation effects in the quantum chaotic scattering, the consideration presented in sections 1-3 are of considerable interest. In particular, the estimate for the scattering amplitudes given by formula (14) is valid for the general case of singular potentials $q(x)$ supported in a compact region D. In this case the corresponding classical repeller Ω_R is generally chaotic. So, the result (14) means that even for classically chaotic (irregular) scattering, the strong quantum correlations in the scattering amplitudes exist in some region of parameters, and are of the

universal nature. The latter means that the quantum correlations in this region of parameters do not depend on the specific character of the resonance structure. The estimate (14) includes the constant c which actually depends on the system's parameters

$$c = c(k_1, k_2, a, \Phi_\nu) \tag{24}$$

That is why it is difficult to establish a relation between the region of parameters where the estimate (14) is valid, and the one ($\delta E > \Gamma > \Delta E$) where the above discussed Ericson fluctuations are important.

The analytical and experimental investigations of the dependence (24) represent a significant interest for the further development of our understanding of the correlation effects in the processes of quantum chaotic scattering.

One of the possibilities to investigate the correlation and fluctuation effects in quantum chaotic scattering can be realized in the microwave experiments (see, for example, [51]). The main idea, which is used in these experiments, is that the Schrödinger equation for a free particle reduces to the Helmholtz equation which describes the propagation of the classical waves. This correspondence was utilized in [51] to investigate the role of fluctuations in the chaotic scattering. In our opinion, this method is rather promising: it allows one to imitate the ballistic regime taking into account scattering, and to study the correlation effects in mesoscopic systems using a microwave technique.

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